

PEARSON
**SPECIALIST
MATHEMATICS**
QUEENSLAND
EXAM PREPARATION WORKBOOK



UNITS 3 & 4

Sample Pages

PEARSON SPECIALIST MATHEMATICS

QUEENSLAND
EXAM PREPARATION WORKBOOK



UNITS 3 & 4

About this Pearson Specialist Mathematics 12 Exam Preparation Workbook

The purpose of the **Pearson Exam Preparation Workbook** is to assist students in their preparation for the QCAA external exams. Answering previous external exam questions is an effective way to do this, as it offers well-constructed questions at the appropriate level.

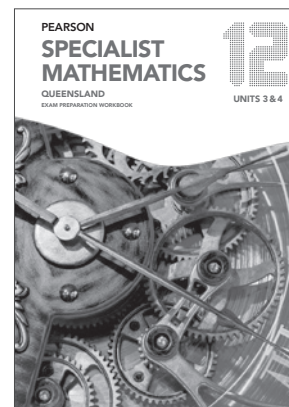
This **Pearson Exam Preparation Workbook** includes previous external exam questions from New South Wales, South Australia and Victoria. Given that both the syllabuses and the access to allowed technologies varies across the states, the author has reviewed questions from years 2000 to 2017 to select those questions that align with the QCAA syllabus.

These questions were then categorised using the QCAA three levels of difficulty: simple familiar, complex familiar and complex unfamiliar. This matches the QCAA external exam structure, which indicates that in Paper 1 and Paper 2, marks will be allocated in the following approximate proportions:

- 60% simple familiar
- 20% complex familiar
- 20% complex unfamiliar.

The source of each question in the **Pearson Exam Preparation Workbook** is referenced at the start of the question. At times, there may be some variation between the notation used in the questions and that used by the QCAA; the authors have made note of this within the worked solution as applicable.

Each worked solution has indicative mark allocations. As official marking schemes are not released by state examining bodies, the mark allocations in the **Pearson Exam Preparation Workbook** are based on the author's and reviewer's on-balance judgement and their teaching experience.



Writing and development team

We are grateful to the following people for their time and expertise in contributing to **Pearson Specialist Mathematics 12 Exam Preparation Workbook**.

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How to use this workbook

Pearson Specialist Mathematics 12 Queensland Exam Preparation Workbook, Units 3 & 4

This exam preparation workbook has been developed to assist students in their preparation for the QCAA external exams. It provides previous external exam questions from New South Wales, South Australia and Victoria that align with the QCAA syllabus. All questions are categorised into the QCAA three levels of difficulty—simple familiar, complex familiar and complex unfamiliar—to match the QCAA external exam structure.

The questions have been grouped into convenient sets, with the intention that each question set is tackled in one sitting.

Levels of difficulty

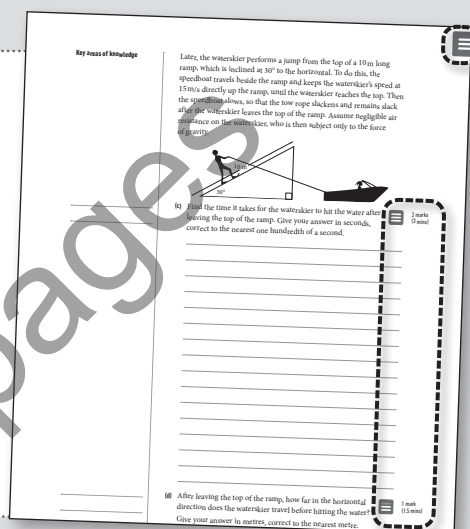
Levels of difficulty are indicated using a three-striped label.

Simple familiar:  Complex familiar: 

Complex unfamiliar: 

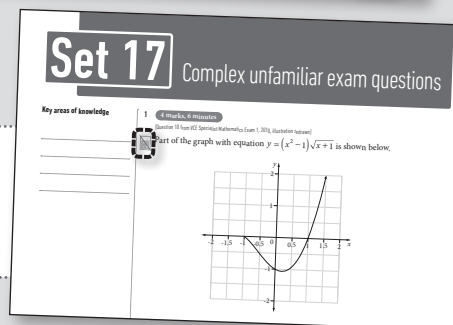
These are used in two ways:

- 1 To show the level of difficulty for the whole question set
- 2 To show the level of difficulty of individual question parts when they differ from that of the question set level. In such cases, all parts of that question are labelled.



Technology free questions

Questions to be completed without the use of any technology will have a crossed out calculator image beside them: 



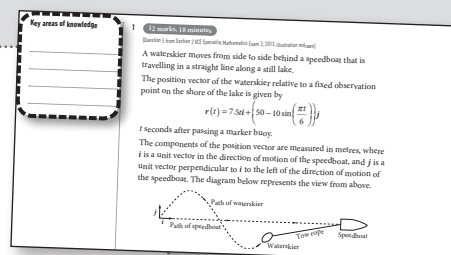
Get yourself exam ready using this 5-step preparation sequence

Step 1: Key areas of knowledge

The purpose of making these notes is to first identify **what** is required to be done, and **how** it might be done, **without** doing it at this stage.

For each question, note the topic(s) of mathematics it draws on, formulas you think will be needed, and any other comments you feel will help you work out the answer.

Then move on to the next question in that set.



Step 2: Complete questions

Complete all the questions within the question set using the space provided.

Question sets have been structured according to level of difficulty, with each question set covering a range of topics from the syllabus.

Key Area of Knowledge

1 **2 marks, 2 minutes**

Question 1 from Section 2, VCE Specialist Mathematics (exam 2015, (suitable revision))

Consider the graph with the rule $|z - i| = 1$ where $z \in \mathbb{C}$.

(a) Write this rule in cartesian form. **2 marks (1 mark)**

(b) Find the points of intersection of the graphs with the rules $|z - i| = 1$ and $|z - 1| = 1$ in cartesian form. **2 marks (1 mark)**

Step 3: Check your answer

Review and mark your answers according to the solutions provided in the corresponding worked solutions.

Set 2 Simple familiar worked solutions and examination report

Examination report comments

Marks	0	1	2	Average
%	14	10	76	1.6

$|x + iy - 1| = 1 \Rightarrow x^2 + y^2 - 2x = 0$

This question was generally quite well done. However, a common error was that students tried to make y the subject and neglect the x sign. Another common error was getting i mixed up in the answer.

1 mark for correctly translating centre
1 mark for correctly translating the radius

Notes and pointers

Worked solutions

(a) In complex form the circle is represented by $|z - i| = r$. The coordinates of the centre and the length of the radius can be determined and substituted into the cartesian form of the circle. An alternative method is to substitute $z = x + yi$ into the given equation, evaluate the modulus, and simplify algebraically.

$|z - i| = r$
 $a_1 = 4, r = 1$

\therefore centre $(1, 0)$, radius is 1
 $(x - 1)^2 + y^2 = 1$

Marks

Step 4: Examination report and reflection

Review the marks obtained from past students, read the information in the **Examination report** section (where available) and reflect on your own solution.

Use the **Notes and pointers** section to write down any relevant key reminders to yourself about common errors, key rules etc.

Set 2 Simple familiar worked solutions and examination report

Examination report comments

Marks	0	1	2	Average
%	14	10	76	1.6

$|x + iy - 1| = 1 \Rightarrow x^2 + y^2 - 2x = 0$

This question was generally quite well done. However, a common error was that students tried to make y the subject and neglect the x sign. Another common error was getting i mixed up in the answer.

1 mark for correctly translating centre
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Notes and pointers

Worked solutions

(a) In complex form the circle is represented by $|z - i| = r$. The coordinates of the centre and the length of the radius can be determined and substituted into the cartesian form of the circle. An alternative method is to substitute $z = x + yi$ into the given equation, evaluate the modulus, and simplify algebraically.

$|z - i| = r$
 $a_1 = 4, r = 1$

\therefore centre $(1, 0)$, radius is 1
 $(x - 1)^2 + y^2 = 1$

Marks

Step 5: Self-reflection: Question set notes and pointers summary

Reflect on all the questions within one set, review your comments in the individual Notes and pointers sections, and use these to complete a summary of the overall question set.

Use the **Red**, **Amber** and **Green** categories to note what you need to revise or don't understand, what you need to watch out for, and what you did well.

Once all sets are completed, these summaries will help in giving you direction on where to focus your further revision.

Self-reflection:
Question set *Notes and pointers* summary

Red

- Most concepts, rules, topics I need to revise to don't understand

Amber

- Common errors I need to make and need to watch out for

Green

- Things I always do well

Set 1

Set 2

Set 3

Set 4

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Key areas of knowledge

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1 **6 marks, 9 minutes**

[Question 1 from Section 2, VCE Specialist Mathematics Exam 2, 2011, illustration redrawn]

Consider the graph with the rule $|z - i| = 1$ where $z \in \mathbb{C}$.

(a) Write this rule in cartesian form.

2 marks
(3 mins)

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(b) Find the points of intersection of the graphs with the rules $|z - i| = 1$ and $|z - 1| = 1$ in cartesian form.

2 marks
(3 mins)

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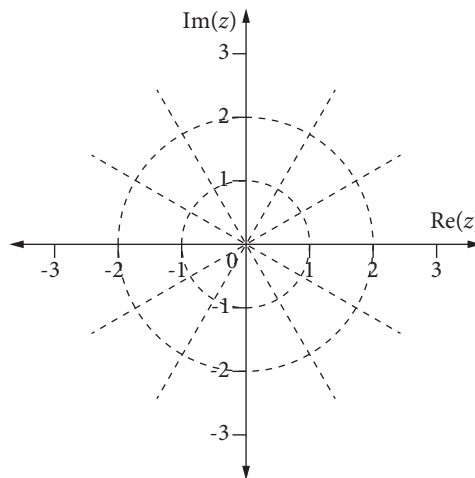
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(c) Sketch and label the graphs with rules $|z - i| = 1$ and $|z - 1| = 1$ on the argand diagram below.

2 marks
(3 mins)



My total marks:



Key areas of knowledge

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2 4 marks, 6 minutes

[Question 9 VCE Specialist Mathematics Exam 1, 2015]



Consider the curve represented by $x^2 - xy + \frac{3}{2}y^2 = 9$.

(a) Find the gradient of the curve at any point (x, y) .

2 marks
(3 mins)

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(b) Find the equation of the tangent to the curve at the point $(3,0)$ and find the equation of the tangent to the curve at the point $(0, \sqrt{6})$.

2 marks
(3 mins)

Write each equation in the form $y = ax + b$.

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My total marks:

3 3 marks, 4.5 minutes

[Question 13c from Section 2, HSC Mathematics Extension 1, 2015]

Prove by mathematical induction that for all integers $n \geq 1$,

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

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My mark:

4 1 mark, 1.5 minutes

[Question 17 from Section 1, VCE Specialist Mathematics Exam 2, 2014]

The acceleration vector of a particle that starts from rest is given by $\underline{a}(t) = -4 \sin(2t)\underline{i} + 20 \cos(2t)\underline{j} - 20e^{-2t}\underline{k}$, where $t \geq 0$.

The velocity of the particle, $\underline{v}(t)$, is given by

- A $-8 \cos(2t)\underline{i} - 40 \sin(2t)\underline{j} + 40e^{-2t}\underline{k}$
- B $2 \cos(2t)\underline{i} + 10 \sin(2t)\underline{j} + 10e^{-2t}\underline{k}$
- C $(8 - 8 \cos(2t))\underline{i} - 40 \sin(2t)\underline{j} + (40e^{-2t} - 40)\underline{k}$
- D $(2 \cos(2t) - 2)\underline{i} + 10 \sin(2t)\underline{j} + (10e^{-2t} - 10)\underline{k}$
- E $(4 \cos(2t) - 4)\underline{i} + 20 \sin(2t)\underline{j} + (20 - 20e^{-2t})\underline{k}$

My mark:

5 1 mark, 1.5 minutes

[Question 20 from Section A, VCE Specialist Mathematics Exam 2, 2016]

The lifetime of a certain brand of batteries is normally distributed with a mean lifetime of 20 hours and a standard deviation of two hours. A random sample of 25 batteries is selected.

The probability that the mean lifetime of this sample of 25 batteries exceeds 19.3 hours is

- A 0.0401 B 0.1368 C 0.6103
- D 0.8632 E 0.9599

My mark:

6 5 marks, 7.5 minutes

[Question 2(a) only from Section 2, VCE Specialist Mathematics Exam 2, 2014]

Consider the complex number $z_1 = \sqrt{3} - 3i$.

(a) (i) Express z_1 in polar form.

2 marks
(3 mins)

(ii) Find $\text{Arg}(z_1^4)$.

1 mark
(1.5 mins)



Key areas of knowledge

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(iii) Given that $z_1 = \sqrt{3} - 3i$ is one root of the equation $z^3 + 24\sqrt{3} = 0$, find the other two roots, expressing your answers in cartesian form.

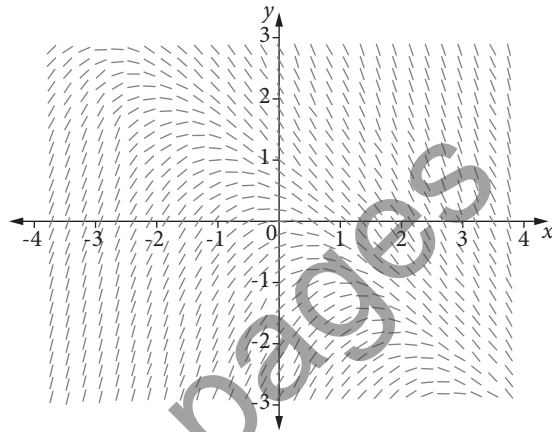
2 marks
(3 mins)

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My total marks:

7 1 mark, 1.5 minutes

[Question 10 from Section A, VCE Specialist Mathematics Exam 2, 2016, illustration redrawn]



The direction field for the differential equation $\frac{dy}{dx} + x + y = 0$ is shown above. A solution to this differential equation that includes $(0, -1)$ could also include

- A $(3, -1)$ B $(3.5, -2.5)$ C $(-1.5, -2)$
- D $(2.5, -1)$ E $(2.5, 1)$

My mark:

8 1 mark, 1.5 minutes

[Question 5 from Section 1, VCE Northern Hemisphere Specialist Mathematics Exam 2, 2017]

Given that A, B, C and D are non-zero rational numbers, the expression $\frac{3x + 1}{x(x - 2)^2}$ can be represented in partial fraction form as

- A $\frac{A}{x} + \frac{B}{(x - 2)}$ B $\frac{A}{x} + \frac{B}{(x - 2)^2}$
- C $\frac{A}{x} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$ D $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x - 2)}$
- E $\frac{A}{x} + \frac{B}{(x - 2)} + \frac{Cx + D}{(x - 2)^2}$

My mark:

Set 2

Simple familiar worked solutions and examination report

Examination report comments

Marks	0	1	2	Average
%	14	10	76	1.6

$$|x + iy - i| = 1 \Rightarrow x^2 + (y - 1)^2 = 1$$

This question was generally quite well done. However, a common error was that students tried to make y the subject and neglect the \pm sign. Another common error was getting i mixed up in the answer.

1 mark for correctly translating centre

1 mark for correctly translating the radius

Notes and pointers

Marks	0	1	2	Average
%	21	9	70	1.5

This question was generally done well. Equivalent forms of points $(0,0)$ and $(1,1)$ were also accepted.

1 mark for correctly identifying $(0,0)$

1 mark for correctly identifying $(1,1)$

Notes and pointers

Worked solutions

Marks

- 1 (a) In complex form the circle is represented by $|z - z_1| = r$. The coordinates of the centre and the length of the radius can be determined and substituted into the cartesian form of the circle. An alternative method is to substitute $z = x + yi$ into the given equation, evaluate the modulus, and simplify algebraically.

$$|z - z_1| = r$$

$$z_1 = i, r = 1$$

1

\therefore centre $(1,0)$, radius is 1

$$(x - 1)^2 + y^2 = 1$$

1

- (b) There are two common approaches to solving this question. First, both equations can be sketched or drawn on a graphics calculator, and the coordinates of the point of intersection can be determined. Second, an algebraic solution can be found. This is shown below. If an algebraic solution is used, graphing on a graphics calculator is an appropriate method to check that results are reasonable.

$$|z - 1| = 1 \Rightarrow x^2 + (y - 1)^2 = 1$$

Equate both circles:

$$(x - 1)^2 + y^2 = x^2 + (y - 1)^2$$

$$x^2 - 2x + 1 + y^2 = x^2 + y^2 - 2y + 1$$

$$-2x = -2y$$

$$x = y$$

Substitute $y = x$ into the equation of the first circle and solve for x :

$$(x - 1)^2 + x^2 = 1$$

$$x^2 - 2x + 1 + x^2 = 1$$

$$2x^2 - 2x = 0$$

$$2(x^2 - x) = 0$$

$$2x(x - 1) = 0$$

$\therefore x = 0$ and $x = 1$

Given $x = y$ the points of intersection are $(0,0)$ and $(1,1)$.

2

Examination report comments

Marks	0	1	2	Average
%	22	16	62	1.4

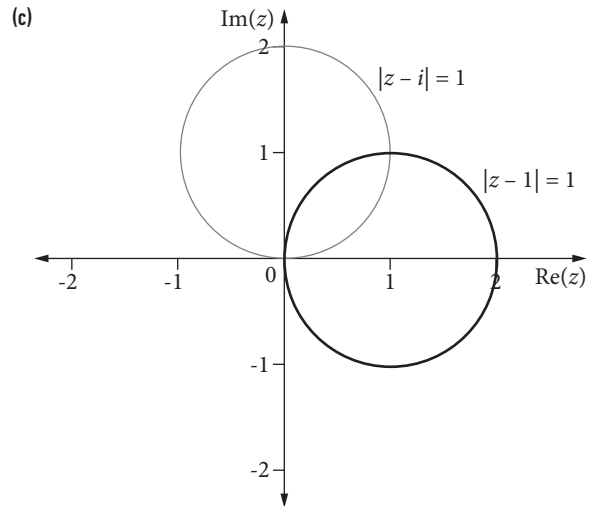
This question was reasonably well done. Common errors included the incomplete labelling of the circles, and circles having wrong centres. A small number of students gave lines or points instead of circles.

- 1 mark for correctly sketching $|z - i| = 1$
- 1 mark for correctly sketching $|z - 1| = 1$

Notes and pointers

Worked solutions

Marks



2

Marks	0	1	2	Average
%	14	16	70	1.6

This question was answered well. Most students correctly used the product and chain rules. A number of sign errors appeared on the left-hand scale, while some left the right-hand side as 9 after differentiating the left-hand scale. Algebraic simplification errors were common.

- 1 mark for correctly identifying and using chain and product rules in implicit differentiation
- 1 mark for the correct answer

Notes and pointers

- 2 (a) It is important to identify where the chain and product rules will apply, and also to note the subtraction sign in front of the second term on the left-hand side.

$$\begin{aligned} \frac{d}{dx} \left(x^2 - xy + \frac{3}{2} y^2 \right) &= \frac{d}{dx} (9) \\ \frac{d}{dx} (x^2) - \frac{d}{dx} (xy) + \frac{d}{dx} \left(\frac{3}{2} y^2 \right) &= 0 \\ 2x - \left(x \frac{d}{dx} (y) + y \frac{d}{dx} (x) \right) + \frac{3}{2} \frac{d}{dy} (y^2) \frac{dy}{dx} &= 0 \\ 2x - x \frac{d}{dy} (y) \frac{dy}{dx} - y + \frac{3}{2} 2y \frac{dy}{dx} &= 0 \\ 2x - x \frac{dy}{dx} - y + \frac{3}{2} 2y \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} \left(\frac{6}{2} y - x \right) &= -2x + y \\ \frac{dy}{dx} &= \frac{y - 2x}{3y - x} \end{aligned}$$

The gradient of the curve at any point (x, y) is

$$\frac{dy}{dx} = \frac{y - 2x}{3y - x} \quad 1$$

Examination report comments

Marks	0	1	2	Average
%	18	20	62	1.5

This question was well answered. The most common errors were only finding one equation and arithmetic errors in converting one or both equations to the required form.

1 mark for finding the first equation to the tangent correctly

1 mark for finding the second equation to the tangent correctly

Notes and pointers

Note: This question is from a NSW HSC examination paper. Examiner reports do not supply a breakdown of marks achieved by the student cohort for these papers.

1 mark for verifying the initial case

1 mark for verifying the initial case and using inductive assumption

1 mark for a complete and correct proof

Notes and pointers

Worked solutions

Marks

- (b) Use the previous answer to determine the gradient of the tangent to the curve at each point.

For point $(3, 0)$

$$m = \frac{0 - 2 \times 3}{3 \times 0 - 3} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - 3)$$

$$y = 2x - 6$$

The equation of the tangent to the curve at the point $(3, 0)$ is $y = 2x - 6$.

1

For point $(0, \sqrt{6})$,

$$m = \frac{\sqrt{6} - 2 \times 0}{3\sqrt{6} - 0} = \frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{6} = \frac{1}{3}(x - 0)$$

$$y = \frac{1}{3}x + \sqrt{6}$$

The equation of the tangent to the curve at the point $(0, \sqrt{6})$ is $y = \frac{x}{3} + \sqrt{6}$

1

- 3 The assumption $n = k$ must be used when proving for $n = k + 1$.

When $n = 1$,

$$\text{RHS} = 1 - \frac{1}{(1+1)!}$$

$$= 1 - \frac{1}{2!} = \frac{1}{2} = \text{LHS}$$

The statement is therefore true for $n = 1$.

1

Assume the result is true for $n = k$

i.e. assume $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$ to be true.

Prove true for $n = k + 1$

i.e.

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{k}{(k+1)!} + \frac{k}{(k+1+1)!} = 1 - \frac{1}{(k+1+1)!}$$

$$\text{LHS} = 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!} \text{ (from assumption)}$$

1

$$= 1 - \frac{(k+2) - (k+1)}{(k+2)!}$$

$$= 1 - \frac{1}{(k+2)!} = \text{RHS}$$

Hence the result is proven by mathematical induction.

1

Examination report comments

% A	% B	% C	% D	% E	% No Answer
6	24	6	62	2	0

The correct option is **D**.

Notes and pointers

% A	% B	% C	% D	% E	% No Answer
5	4	14	7	68	1

The correct option is **E**.

$$E(\bar{X}) = 20, \text{sd}(\bar{X}) = \frac{2}{\sqrt{25}} = \frac{2}{5}$$

Notes and pointers

Marks	0	1	2	Average
%	3	17	80	1.8

Overall, most students answered this question well, but incorrect arguments such as $\frac{\pi}{3}$ or $\frac{\pi}{6}$ were common. The argument $\frac{5\pi}{3}$ was also accepted.

1 mark for the correct modulus

1 mark for the correct argument written in polar form

Notes and pointers

Worked solutions

Marks

- 4 Integrate acceleration to get the velocity equation.

$$\begin{aligned} v(t) &= \int a(t) dt \\ &= \int (-4 \sin(2t)\mathbf{i} + 20 \cos(2t)\mathbf{j} - 20e^{-2t}\mathbf{k}) dt \\ &= 2 \cos(2t)\mathbf{i} + 10 \sin(2t)\mathbf{j} + 10e^{-2t}\mathbf{k} + \mathbf{c} \end{aligned}$$

Solve $v(0) = 0$ to determine the value of the constant of integration.

$$\begin{aligned} 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} &= 2 \cos(2t)\mathbf{i} + 10 \sin(2t)\mathbf{j} + 10e^{-2t}\mathbf{k} + \mathbf{c} \\ \mathbf{c} &= -2\mathbf{i} - 10\mathbf{k} \end{aligned}$$

$$\therefore v(t) = (2 \cos(2t) - 2)\mathbf{i} + 10 \sin(2t)\mathbf{j} + (10e^{-2t} - 10)\mathbf{k} \quad 1$$

- 5 This probability is calculated using a cumulative density function together with lower boundary of 19.3, upper boundary of ∞ , standard deviation and mean.

$$E(\bar{X}) = 20$$

$$\begin{aligned} \text{sd}(\bar{X}) &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{2}{\sqrt{25}} \\ &= \frac{2}{5} \\ &= 0.4 \end{aligned}$$

$$P(X > 19.3) = 0.9599 \quad 1$$

- 6 (a) (i) Sketching the point z_1 on an Argand diagram will help ensure your answer is reasonable, particularly the angle as it is located in the fourth quadrant.

$$\begin{aligned} |z_1| &= \sqrt{(\sqrt{3})^2 + (-3)^2} \\ &= \sqrt{12} \\ &= 2\sqrt{3} \end{aligned} \quad 1$$

$$\begin{aligned} \text{Arg}(z_1) &= \tan^{-1}\left(\frac{-3}{\sqrt{3}}\right) \\ &= -\tan^{-1}\left(\frac{3}{\sqrt{3}}\right) \\ &= -\frac{\pi}{3} \end{aligned}$$

$$\therefore z_1 = 2\sqrt{3} \text{cis}\left(-\frac{\pi}{3}\right) \quad 1$$

Examination report comments

Marks	0	1	Average
%	33	67	0.7

This question was answered reasonably well, but many students did not express their angle in the interval $(-\pi, \pi]$. A number of students gave the entire expression for $(z_1)^4$ as an answer.

Notes and pointers

Marks	0	1	2	Average
%	14	20	66	1.6

This question was answered reasonably well. Most students could find the conjugate root, but a number could not obtain $-2\sqrt{3}$, often omitting the negative sign. A number of students gave factors instead of the roots.

1 mark for first root correct

1 mark for second root correct

Notes and pointers

% A	% B	% C	% D	% E	% No Answer
6	65	14	11	4	0

The correct option is **B**.

Notes and pointers

Worked solutions

Marks

- (ii) Recall that $(r \operatorname{cis}(\theta))^n = r^n \operatorname{cis}(n\theta)$. When working with angles, ensure your answer is a principle argument.

$$\begin{aligned} \operatorname{Arg}(z_1^4) &= 4 \times \operatorname{Arg}(z_1) \\ &= 4 \times -\frac{\pi}{3} \\ &= -\frac{4\pi}{3} \\ &= \frac{2\pi}{3} \end{aligned}$$

1

- (iii) There are three roots because the polynomial is cubic. The roots all have the same modulus and are equidistant from each other in angle.

Therefore these roots are $\frac{2\pi}{3}$ apart.

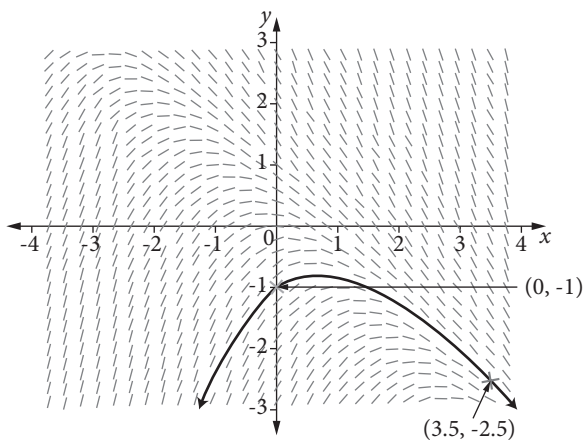
$$\begin{aligned} z_2 &= 2\sqrt{3} \operatorname{cis}\left(-\frac{\pi}{3} + \frac{2\pi}{3}\right) \\ &= 2\sqrt{3} \operatorname{cis}\left(\frac{\pi}{3}\right) \\ &= \sqrt{3} + 3i \end{aligned}$$

1

$$\begin{aligned} z_3 &= 2\sqrt{3} \operatorname{cis}\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) \\ &= 2\sqrt{3} \operatorname{cis}(\pi) \\ &= -2\sqrt{3} \end{aligned}$$

1

- 7 Locate the given point $(0, -1)$ on the gradient field and trace either direction using the gradient field. Check each option to see which best fits on the drawn line.



A solution that includes $(0, -1)$ also includes $(3.5, -2.5)$.

1



Examination report comments

Note: This question is from a Northern Hemisphere examination paper. Examiner reports do not supply a breakdown of marks achieved by the student cohort for these papers.

The correct option is **C**.

Notes and pointers

Worked solutions

8 The denominators for this partial fraction must be x , $(x - 2)$ and $(x - 2)^2$ because a linear term to a power must be shown, with each power starting at 1 until the power indicated, in this case 2.

This leaves only options C and E. Since the quadratic denominator is a perfect square, the numerator must be a constant. This means option C is the correct option.

Marks

1

Sample pages